CSCI 3210: Computational Game Theory

Intro to Computational Social Choice (COMSOC): Fairness

Handbook of COMSOC
Ch 11, 13

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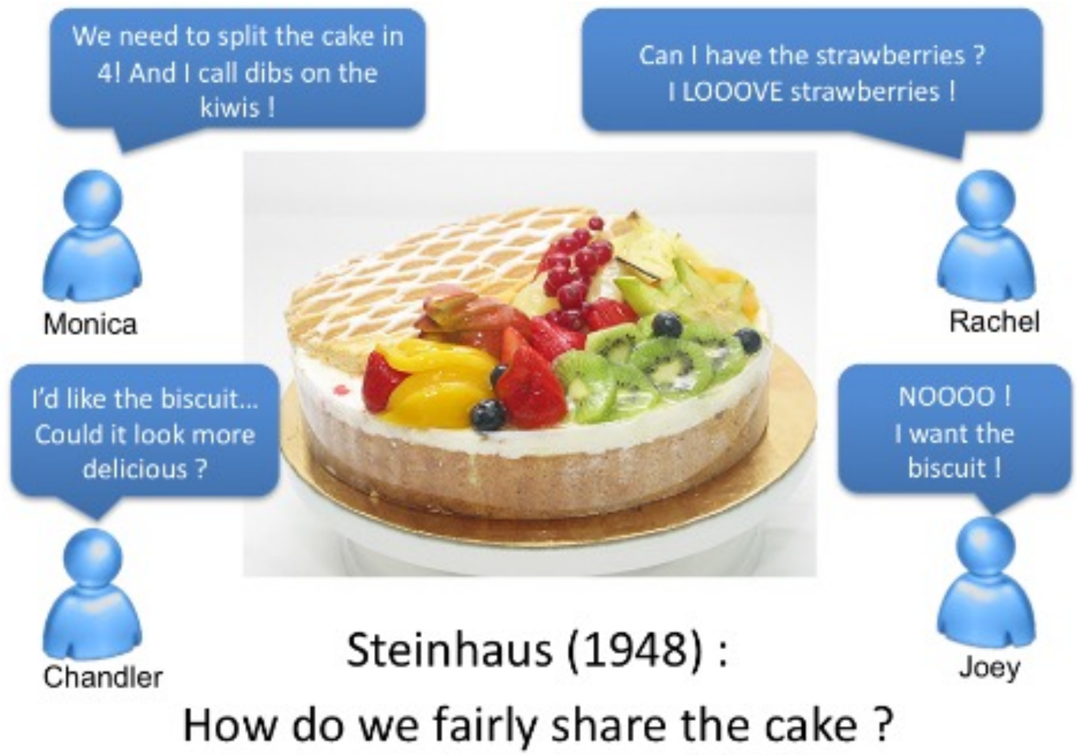
How to best reconcile conflicting interests of a set of agents?

Cake Cutting Algorithms

Ch 13

Cake cutting problem

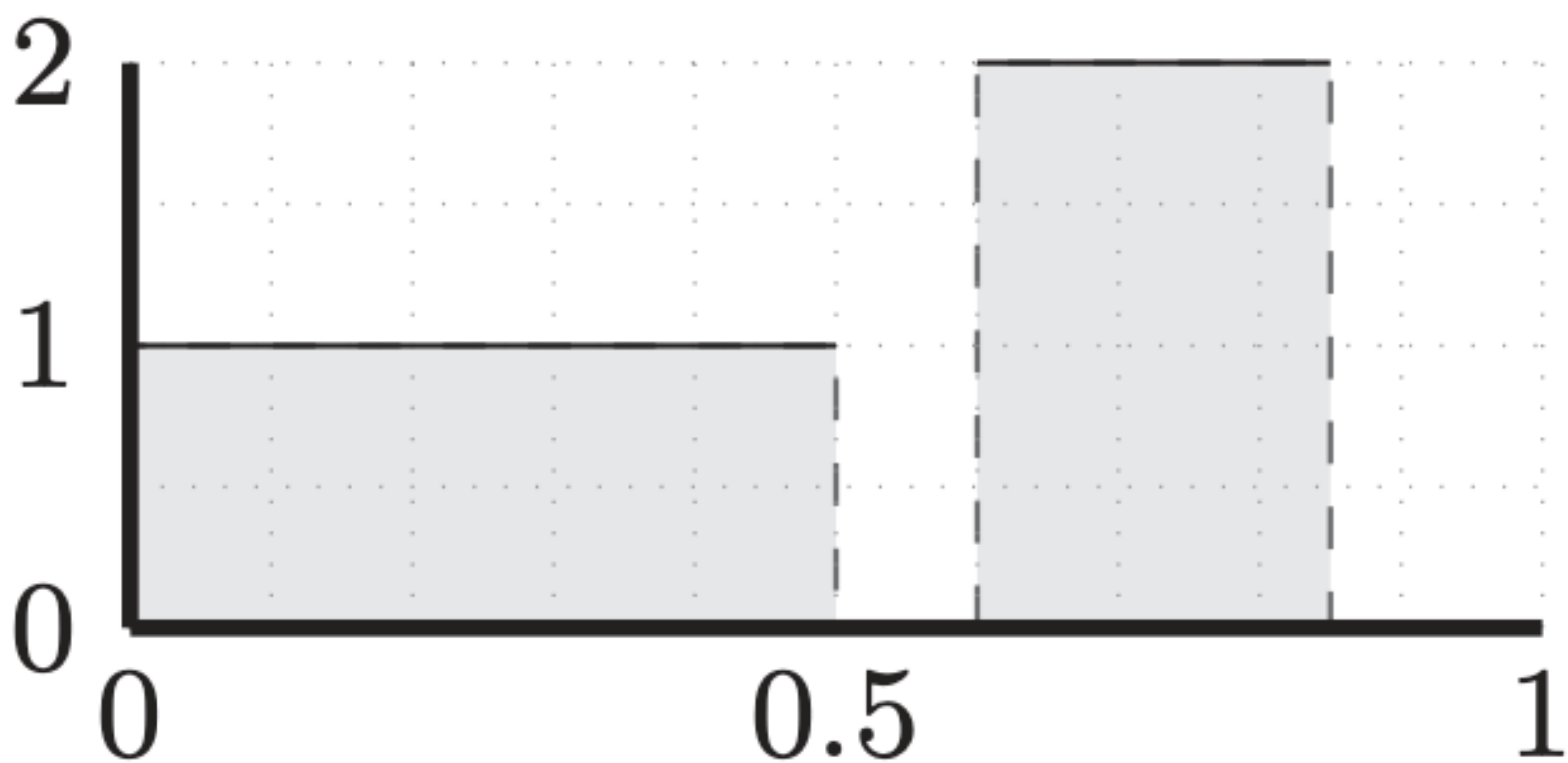


- Existence vs computation
- Individual fairness vs social welfare

http://www.science4all.org/article/fair-division/

Model

- Cake, a heterogeneous divisible good: [0, 1]
- n agents
- Agent i's *private* valuation for interval I: $V_i(\mathrm{I}) \geq 0$
  - Value of the whole cake = 1
  - Valuation is additive


2
1
0
0
0.5
1

Cake cutting problem

- Partition the cake into n pieces: $A_1, A_2, \ldots, A_n$
  - Allow non-contiguous piece?
- One piece for each agent: Agent i gets A_i
- Satisfy some fairness criteria

Fairness Criteria

Envy-freeness

Each agent weakly prefers their piece to any other's

For all $i, j \in N$, $V_i(A_i) \geqslant V_i(A_j)$

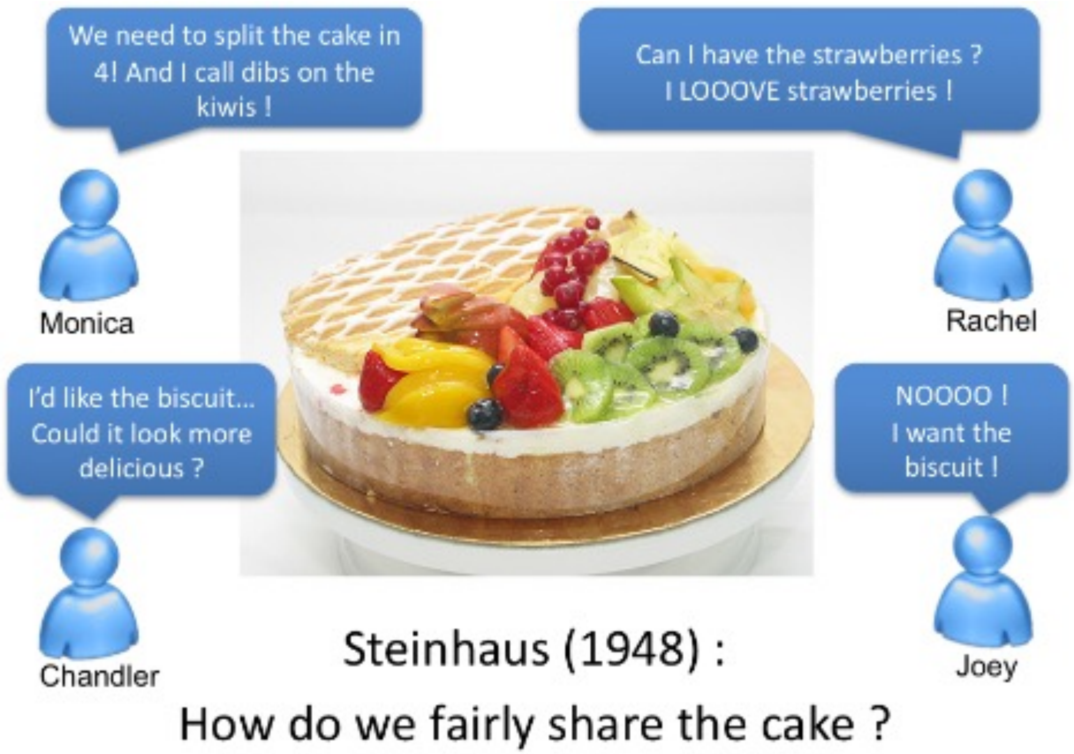
Equitability

Every agent gets the same value

For all $i, j \in N$, $V_i(A_i) = V_j(A_j)$

Equitable doesn't mean envy-free!

- Example: Assign everyone what they <u>don't</u> want
- Everyone gets a value of 0 (equitable) but is envious of some other agent



Proportionality

Each agent gets a value of at least 1/n

for all $i \in N$, $V_i(A_i) \geqslant 1/n$

Envy-freeness implies proportionality

- Suppose agent i is envy-free
- i's value for the whole cake is 1:

$$\sum_{j \in N} V_i(A_j) = 1$$

- So, there must be a piece with value >= 1/n for i
- If i gets a piece of value < 1/n, i will be envious

Converse is true for 2 agents only

So, envy-freeness is stronger than proportionality

Existence Results

Envy-free AND equitable

- It is possible to cut the cake in $n^2 - n$ places and partition the intervals into n pieces (potentially non-contiguous) such that the value is 1/n no matter who gets which piece.
(Alon, 1987)
- Pro: Polynomial # of cuts
- Con: Exists but impossible to compute!

Algorithmic Results

Envy-free for n = 2: cut-and-choose

- Agent 1 cuts the cake into two equally valued pieces (to her)
  - Proportional and envy-free (immune to agent 2)
- Agent 2 chooses a piece
  - Proportional and envy-free, even if agent 1 didn't cut evenly for herself

Envy-free for n = 3: Selfridge-Conway (1960)

1. Agent 1 makes 3 equally valued pieces (to her)

Then what?

(figure out with your peers)

Complexity of envy-free

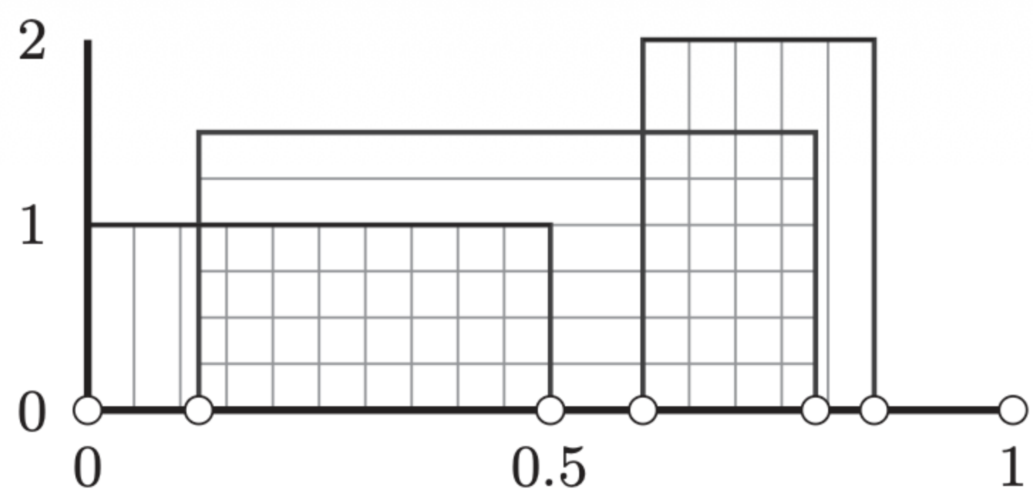
- For 3 or more agents, there is no finite envy-free cake-cutting algorithm that outputs contiguous allocations.
  (Stromquist, 2008)
- Any envy-free cake-cutting algorithm needs $\Omega(n^2)$ operations (Non-contiguous pieces are OK).
  (Procaccia, 2009)

Socially optimal cake cutting

- Social welfare = sum of valuations
- Maximize social welfare with fairness constraints
- Assume valuations are known to the algorithm

Socially optimal cake cutting

- Example: 2 agents
- Piecewise constant valuations: Horizontal vs. vertical lines for agents 1 and 2
- White circles: Intervals of interest



Every agent's valuation is constant in any interval!

LP: Socially optimal proportional allocation

$$\max \sum_{i=1}^{n} \sum_{I \in \mathcal{J}} f_{iI} V_i(I),$$

Maximize social welfare

$$\text{s.t.} \sum_{i=1}^{n} f_{iI} \leqslant 1 \; \forall I \in \mathcal{J},$$

Allocations must be fractions

$$\sum_{I \in \mathcal{J}} f_{iI} V_i(I) \geqslant \frac{1}{n} \; \forall i \in N,$$

Proportionality constraint

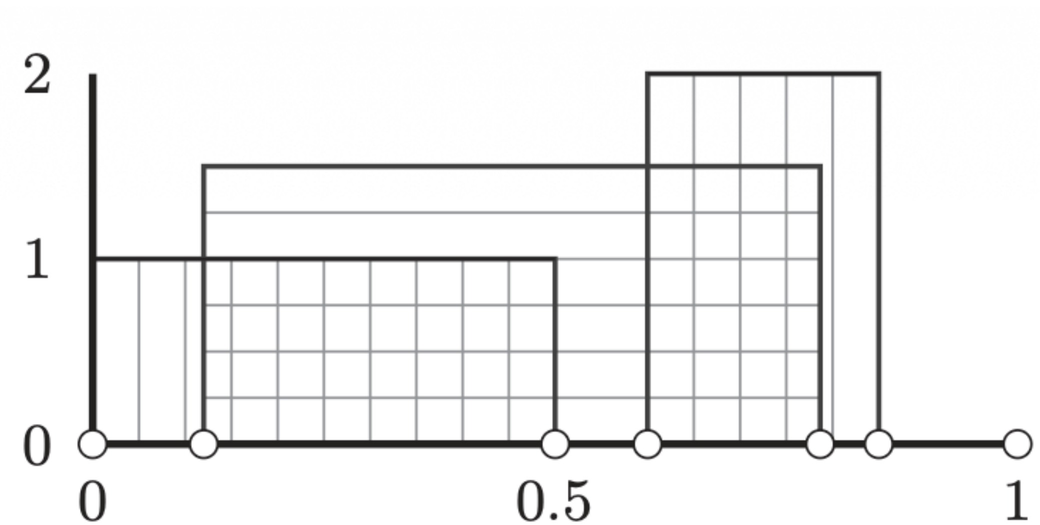
$$f_{iI} \geqslant 0 \; \forall i \in N, I \in \mathcal{J}.$$

Allocations must be >= 0

Similar LPs exist for other fairness constraints

Equitable vs. envy-free: Which is socially better?

- Answer: Envy-free is socially more desirable (Brams, 2012)
- Proof: Using linear Fisher market!



Each interval of white circles is a good in Fisher's market

So, what is fairness?

Does it mean equality?

- Equality may not be feasible
  - Ex: assigning dorm rooms– everyone wants to get into the best one

- Equality may not be Pareto efficient
  - Ex: single-peaked preference

Fairness

- Inequality
- Recognizing differences among the agents and treating different agents differently
- Recognizing and measuring departures from fairness
- Axiomatic approach